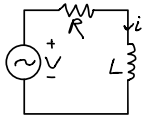


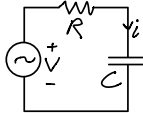
AC Circuits & Phasors

General RLC Circuits



$$E_L = \frac{Li^2}{2}$$

- stores energy from changing current in a magnetic field
- current **lags** behind voltage: $\phi_v > \phi_i$



$$E_C = \frac{Cv^2}{2}$$

- stores energy from changing voltage in an electric field
- current **leads** voltage: $\phi_v < \phi_i$

Phasors

- uses a **single complex number** to describe the **phase & amplitude** of a sinusoid
- all vectors rotate at same frequency (ω)

$$A \sin(\omega t \pm \theta) \rightarrow A \angle \pm \theta$$

- Impedances: $Z_L = j\omega L = jX_L$ $Z_C = \frac{-j}{\omega C} = -jX_C$ $Z_R = R$

Root-Mean-Square

- the AC value which provides the same effect as a steady DC value

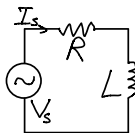
$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

Qualitative Phasor Analysis

- use phasors (vectors) to represent voltage and/or current equations

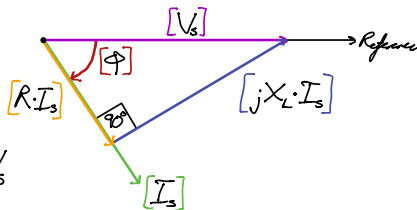
Ex: RL Circuit



$$V_s = R \cdot I_s + j\omega L \cdot I_s = R \cdot I_s + jX_L \cdot I_s$$

$$\Phi = \arctan\left(\frac{X_L}{R}\right)$$

- I_s **lags** Φ behind V_s



Power

- active** power is consumed by linear components (R)
- reactive** power is stored in electric & magnetic fields of sinusoidal components (L/C)

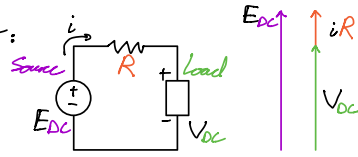
$$p(t) = v(t) \cdot i(t) = V_{rms} \cdot I_{rms} [\cos \Phi + \cos(2\omega t + \Phi)]$$

- average power per cycle: $P = \frac{1}{T} \int_{t-T}^t p(t) dt$

- Power Factor Correction: implement inductors/capacitors to decrease **reactive power**

Transmitting Electric Power

DC Power:



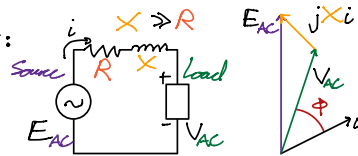
- power will transfer from side with higher voltage
- loss will be difference in voltage

$$P = E_{DC} \cdot i$$

$$P = V_{DC} \cdot i$$

$$P_{loss} = (E_{DC} - V_{DC})i = i^2 R$$

AC Power:



- power will transfer from side with leading voltage

$$S = V_{AC} \cdot i$$

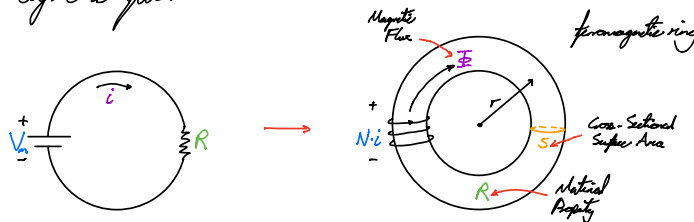
$$P = V_{AC} \cdot i \cdot \cos \phi$$

$$Q = V_{AC} \cdot i \cdot \sin \phi$$

$$P_{loss} = R \cdot i^2$$

Magnetic Circuits

- A magnetic circuit is made up of one or more closed loop paths containing magnetic flux



- Magnetomotive Force ($V_m = Ni$) [Amp-Turns]

- Wrap current carrying coil around magnetically permeable material
- Insert permanent magnet into ring

- Reluctance (R) [$1/H$]

- Flow of flux around magnetic circuit is resisted by reluctance of magnetic circuit

$$R = \frac{L}{\mu S}$$

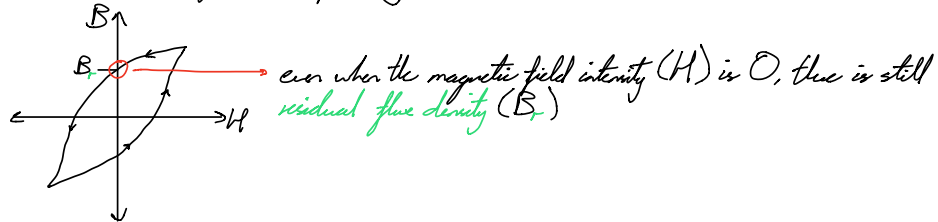
L ← Length of material
 μ ← Magnetic Permeability
 S ← Cross-sectional Surface Area

- Flux flows through path of least reluctance

- Magnetic Flux (Φ) [Wb]

- Ohm's Law applied to magnetostatics: $V_m = \Phi \cdot R$

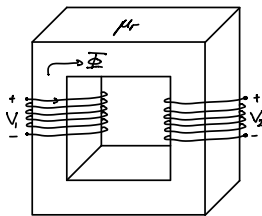
- Hysteresis: Magnetic Flux Density lags behind Magnetic Field Intensity
 - Newly realigned dipoles do not all return to their previous arbitrary directions
 - Hysteresis loop: The relationship between B & H as material is magnetized and demagnetized repeatedly



- Hysteresis loss: Every cycle of the Loop results in energy lost to heat due to the friction generated from dipole movement

Transformers

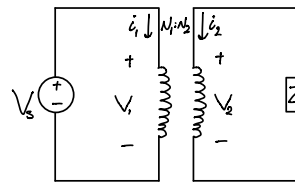
- a passive element which transfers electrical energy from one circuit to another
- Step-Up transformer: $V_2 > V_1$; $i_2 < i_1$
- Step-Down transformer: $V_2 < V_1$; $i_2 > i_1$



$$V_1 = N_1 \frac{d\Phi}{dt}$$

$$V_2 = N_2 \frac{d\Phi}{dt}$$

$$\text{turns ratio: } a = N_1/N_2$$

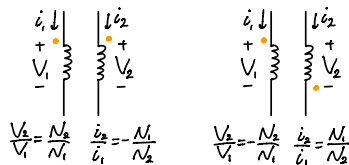


$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$\frac{i_2}{i_1} = -\frac{N_1}{N_2}$$

- H-X Convention
 - 'H' marks the high voltage winding
 - 'X' marks the low voltage winding

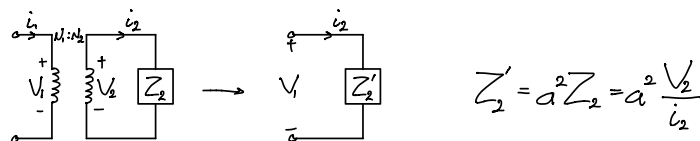
- Dot Convention: indicates polarity of terminals



$$\text{Dots on same side: } \hat{V}_1 = \hat{V}_2 \quad \hat{i}_1 = -\hat{i}_2$$

$$\text{Dots on opposite side: } \hat{V}_1 = -\hat{V}_2 \quad \hat{i}_1 = \hat{i}_2$$

- Impedance Transfer
 - output impedance will appear as Z_2' from the primary's perspective:

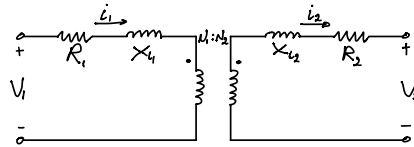


$$Z_2' = a^2 Z_2 = a^2 \frac{V_2}{i_2}$$

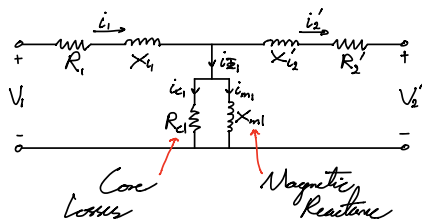
• this applies for transferring primary impedance to secondary: $Z'_1 = a^{-2} Z_1 = a^{-2} \frac{V_1}{i_1}$

T-Circuit Equivalent

• a non-ideal transformer has imperfections that can be modelled:



referred to Primary:



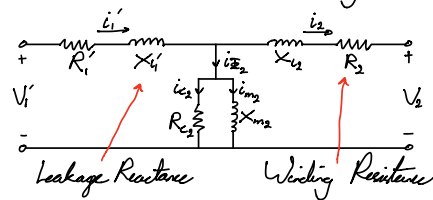
$$V_2' = V_2 a$$

$$X_{2'}' = X_{2'} a^2$$

$$i_2' = \frac{i_2}{a}$$

$$R_2' = R_2 a^2$$

referred to Secondary:



$$V_1' = \frac{V_1}{a}$$

$$X_{1'}' = \frac{X_{1'}}{a^2}$$

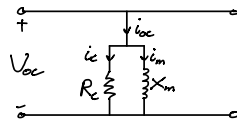
$$i_1' = i_1 \cdot a$$

$$R_1' = \frac{R_1}{a^2}$$

Parameter Determining Tests

Open Circuit Test

• apply voltage across either winding and **open circuit** the other to find the equivalent circuit parameters and losses



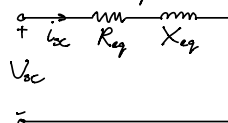
$$V_{oc} = i_c \cdot R_c = i_m \cdot X_m$$

$$P_{oc} = i_c^2 \cdot R_c$$

$$i_{oc}^2 = i_m^2 + i_c^2$$

Short Circuit Test

• apply voltage across either winding and **short circuit** the other to find the equivalent circuit parameters and losses



$$\frac{V_{sc}}{i_{sc}} = Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2}$$

$$P_{sc} = i_{sc}^2 \cdot R_{eq}$$

Voltage Regulation

• measure of how well a transformer can maintain its **secondary** voltage

$$VR = \frac{|V_{2 \text{ no load}}| - |V_{2 \text{ load}}|}{|V_{2 \text{ load}}|}$$

- Efficiency of a transformer is described by: $\eta = \frac{P_{out}}{P_{in}} = \frac{V_2 i_2 \cos \theta_2}{V_2 i_2 \cos \theta_2 + \underbrace{P_c}_{\text{core loss}} + \underbrace{i_1^2 R_1 + i_2^2 R_2}_{\text{copper loss}}}$
- maximum efficiency occurs when $P_c = P_{cu}$

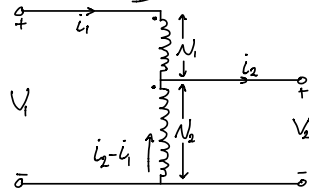
• the output power required for this is: $P_{out} = P_{rated} \cdot \sqrt{\frac{P_c}{P_{cu}}}$

- Transformer Rating (i.e: P_R kVA, V_1/V_2 volts)

- $P_R \rightarrow$ each winding is rated for P_R kVA; $i_1 = P_R/V_1$, $i_2 = P_R/V_2$
- $V_1/V_2 \rightarrow$ the voltage of each of the two windings

- Autotransformer

- the secondary is taken from a tap on the winding



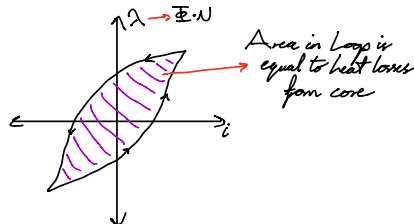
- secondary voltage (V_2) can be adjusted within: $0 < V_2 < V_1$

- rated power in this configuration is: $P_{auto} = V_2 i_2$

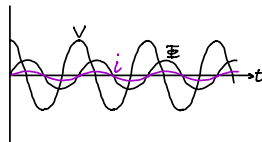
- the tap location can be variable to change the winding ratio: N_1/N_2

- Harmonics

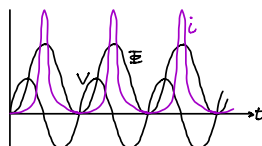
- unwanted frequencies in power systems generated by non-linear conversion of AC to DC
- transformers experience this due to non-linear behaviour of **hysteresis**



- Inrush Current: **spike** in current at transient immediately after switch on
- discharged capacitors offer low impedance allowing for high current to flow



- switching the transformer on when voltage is **maximum** **minimizes inrush current**

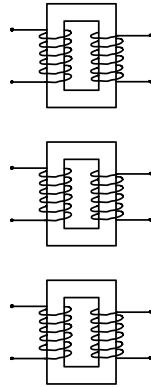


- switching the transformer on when voltage is **zero** **maximizes inrush current**

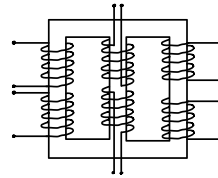
- inrush current decays rapidly due to winding resistance after transient

Three-Phase Transformers

- three single-phase transformers combined to transform bulk electricity

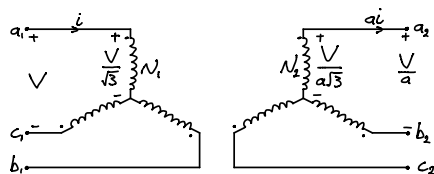


- 3 single-phase cores
- no magnetic coupling

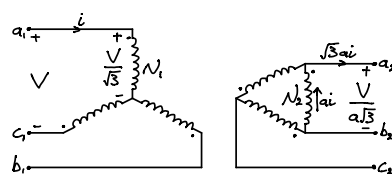


- 3-phase core
- less material use

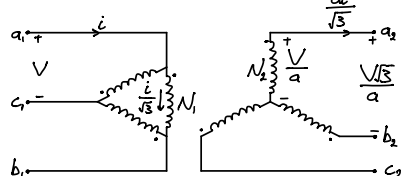
Y-Y connection



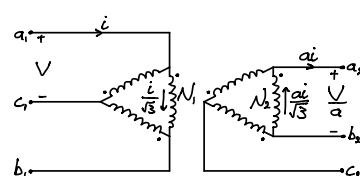
Y-Δ connection



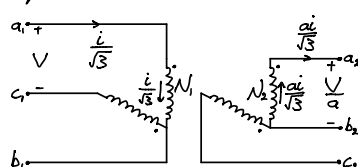
Δ-Y connection



Δ-Δ connection



Open Δ connection



- current decreases ($i \rightarrow \frac{i}{\sqrt{3}}$) when one transformer is removed from Δ-Δ connection

Single-Phase Equivalent Circuit

- examine one phase to determine other phase values if all connections are Y

Δ to Y turn ratio conversion: $a' = \frac{\text{Primary line-line voltage}}{\text{Secondary line-line voltage}} = \sqrt{3}a$

↑ Primary Δ → Y

- Per-Unit System

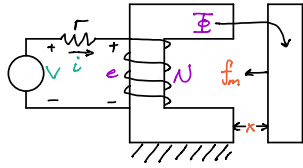
- rescale each side of transformer to create a 1:1 winding ratio

$$\text{Quantity in PU} = \frac{\text{Actual Quantity} \leftarrow \text{value in circuit}}{\text{Base Quantity} \leftarrow \text{selected value}}$$

- this simplifies transformer calculations

Electromechanical Energy Conversion

- energy in system exists in the **Electrical input**, **Mechanical output** and **Coupling field**

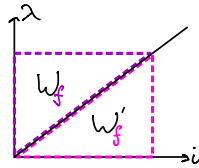


$$W_e = W_f + W_m$$

$$\int e \cdot i dt = W_f + \int f_m dx$$

- Field Energy

- when x is held constant: $W_e = W_f$



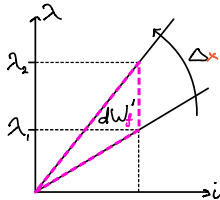
- λ - i relationship can be used to find **energy** and **coenergy**

$$W_f = \int_0^i i d\lambda$$

$$W_f' = \int_0^i \lambda di$$

- Mechanical Force (f_m)

- when x decreases **slowly**:



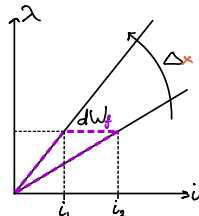
- mechanical work** is the increase in **coenergy**:

$$dW_e = \int_{\lambda_1}^{\lambda_2} i d\lambda$$

$$dW_m = dW_f'$$

$$f_m = \frac{\partial W_f'(i, x)}{\partial x}; i \text{ is constant}$$

- when x decreases **quickly**:



- mechanical work** is the decrease in **field energy**:

$$dW_m = -dW_f$$

$$f_m = -\frac{\partial W_f(\lambda, x)}{\partial x}; \lambda \text{ is constant}$$

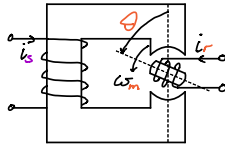
- Linear System

- when **core** reluctance is negligible compared to **air gap** reluctance:

$$\lambda = L(x)i \rightarrow W_f = W_f' = \frac{L(x)i^2}{2} \rightarrow f_m = \frac{dL(x)}{dx} \cdot \frac{i^2}{2} = \frac{B^2}{2\mu_0} \cdot S$$

Rotating Machines

- uses rotational motion to transfer energy from a **stator** to a **rotor**

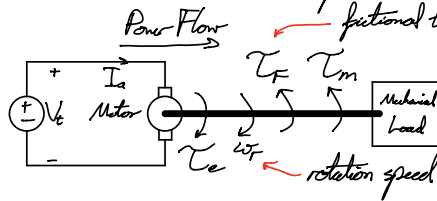


$$W_f = \frac{1}{2} [L_{ss}i_s^2 + L_{rr}i_r^2 + L_{sr}i_s i_r]$$

$$\text{Torque: } \tau_m = \frac{\delta W_f(\lambda, \theta)}{\delta \theta} = \frac{\delta W_f'(i, \theta)}{\delta \theta}$$

DC Machines

- Motor: a machine that inputs rotational motion

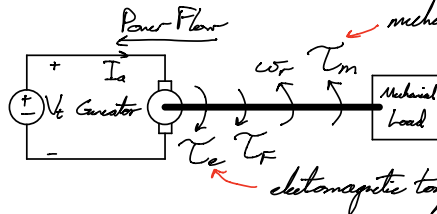


$$P_{out} = \omega_r \tau_m = P_m$$

$$V_t I_a = P_{in}$$

$$\tau_e = \tau_f + \tau_m$$

- Generator: converts rotational motion into electrical power

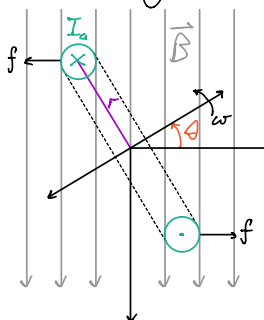


$$P_{in} = \omega_r \tau_m = P_m$$

$$V_t I_a = P_{out}$$

$$\tau_m = \tau_f + \tau_e$$

- Current-Carrying **conductor** in **Magnetic Field** creates **Mechanical Force** → **Lorentz Force**

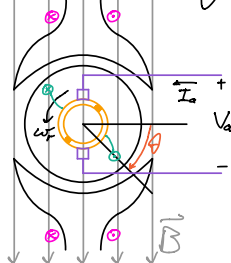


$$f = I_a L B$$

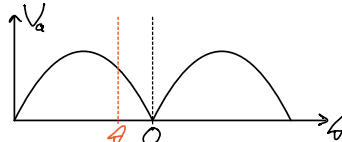
$$\tau_e = 2r L B I_a \cos(\theta) = \Phi_p I_a \cos(\theta) ; \theta = \omega t$$

$$e = \omega \Phi_p \cos(\omega t)$$

- Commutator: rotary electrical switch that periodically reverses current between rotor and circuit



- brushes make contact with **commutator** to allow **current** to alternate after each pass over the **commutator's gaps**
- voltage (V_a) is 0 when $\theta = 0$; brushes contact the **commutator gaps**



- Construction

- Field Winding: use *secondary windings* to generate magnetic field ($\vec{B}$)
- Permanent Magnet: use permanent magnets to generate magnetic field ($\vec{B}$)

- Induced Voltage and Torque

- armature characteristics* depend on physical properties of armature
 $\rightarrow$ this can be represented by K_a ↗ machine armature constant

$$e_a = K_a \Phi_p \omega_r = L_{af} I_f \omega_r$$

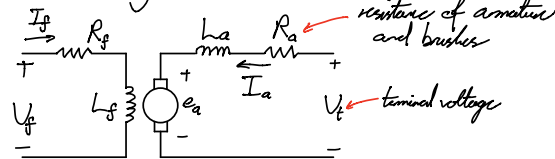
$$\tau_e = K_a \Phi_p I_a = L_{af} I_f I_a$$

↖ back emf

$$e_a I_a = P_e = \omega_r \tau_e$$

- Equivalent Circuit

- Field Winding:

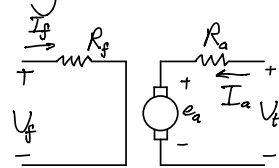


$$P_{in} = V_t I_a$$

$$V_f = R_f I_f + L_f \frac{dI_f}{dt}$$

$$V_t = \pm R_a I_a + e_a + L_a \frac{dI_a}{dt}$$

- Steady-State



$$V_f = R_f I_f$$

$$V_t = \pm R_a I_a + e_a$$

$$\tau_e = \tau_m + K_F \omega_r$$

- Mechanical Motion: combined moment of inertia of motor and load

$$J \frac{d\omega}{dt} = \tau_e - \tau_m - \tau_F$$

- Motor Constants

- Velocity (K_v): ratio of motor's unloaded rotational speed to back emf (e_a)
- Torque (K_τ): ratio of torque to armature current (I_a)

$$\text{Field Winding: } e_a = K_a \frac{L_f}{N_f} I_f \omega_r \quad \tau_e = K_a \frac{L_f}{N_f} I_f I_a$$

$L_{af} \rightarrow$ mutual inductance between field and armature coils

$$\text{Permanent Magnets: } e_a = K_b \omega_r$$

$$\tau_e = K_\tau I_a$$

$$1/K_v = K_b = K_\tau = K_a \Phi_p = K_m \sqrt{R_a}$$

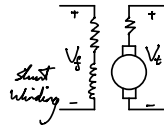
↖ back emf constant

↖ motor constant

• DC Machine Classification

• Separately Excited DC Machine

- field winding is independent of armature

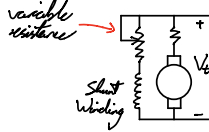


$$e_a = V_t - I_a R_a = K_a \Phi_p \omega_r$$

$$V_t = I_f R_f$$

• Shunt Winding

- field winding supply voltage (V_f) equals armature terminal voltage (V_t)

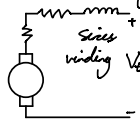


$$e_a = V_t - I_a R_a$$

$$V_t = I_f R_f$$

• Series Winding

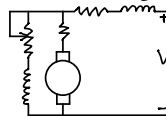
- field winding is connected in series to armature



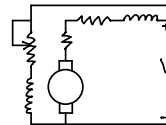
$$e_a = V_t - I_a (R_a + R_{sw})$$

• Compound Machines

- field winding is split between a series and shunt winding



Short-Shunt Compound



Long-Shunt Compound

$$\Phi_{\text{total}} = \Phi_{\text{shunt}} + \Phi_{\text{series}}$$

$$\Phi_{\text{total}} = \Phi_{\text{shunt}} - \Phi_{\text{series}}$$

• Parameter Testing

- Locked Rotor $\rightarrow \omega_r = 0, e_a = 0$

$$R_f = \frac{V_f}{I_f} \quad R_a = \frac{V_t}{I_a}$$

- No Load for Permanent Magnet Motors

$$P_e = e_a I_a = \omega_r \tau_e \rightarrow \tau_e = \tau_p(\omega_r) = K_f \omega_r$$

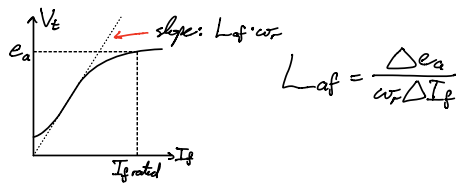
measure friction as a function of speed

- Stopping Transient for Permanent Magnet Motors $\rightarrow$ determines inertia

measure $V_t(t) \rightarrow$ the time it takes to go from $V_t = e_a$ to $V_t = 0; \Delta t$

$$\tau_F(\omega_r) = -J \frac{\Delta e_a}{K_b \Delta t}$$

- Open Circuit $\rightarrow \omega_r$ is constant (rated nominal speed of motor)
- determine mutual inductance between armature and field winding



- Characteristics
 - Speed Regulation = $\frac{\omega_r \text{ no load} - \omega_r \text{ load}}{\omega_r \text{ load}}$

- Voltage Regulation = $\frac{V_{oc} - V_{t \text{ load}}}{V_{t \text{ load}}}$

- Power Efficiency and Flow
 - multiple sources of power loss in motor

$$P_{in} = P_e \quad P_{out} = P_m \quad \rightarrow \quad \eta = \frac{P_{out}}{P_{in}} = \frac{P_m}{P_e}$$

- Speed-Torque Relationship

Separately:
Excited Motor: $\omega_r = \frac{V_t}{K_a \Phi_p} - \tau \frac{R_a}{(K_a \Phi_p)^2}$



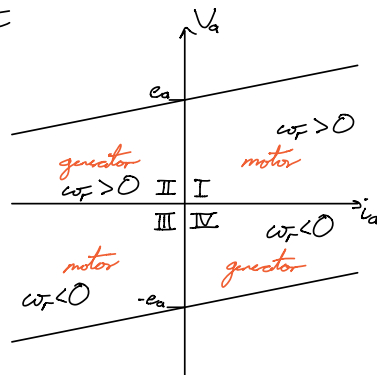
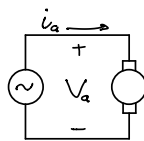
- an increase in torque (τ) causes a decrease in speed (ω_r)

DC Motor Drives and Dynamics

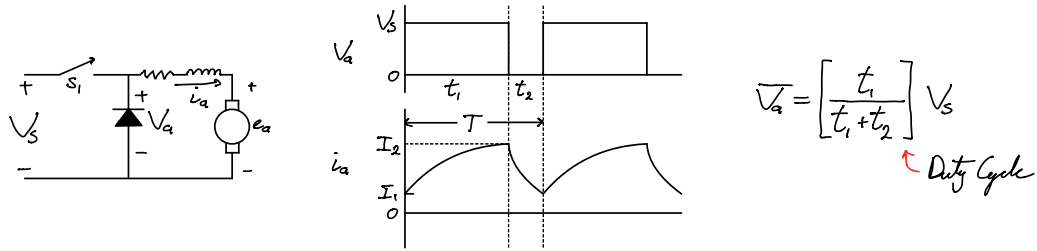
- basic principles of controlling DC motors
- Pulse Width Modulation: allows control of average voltage through periodic pulses
 - used to control voltage sources

average voltage: $\bar{V}_a = \frac{1}{T} \int_0^T V_a(t) dt$

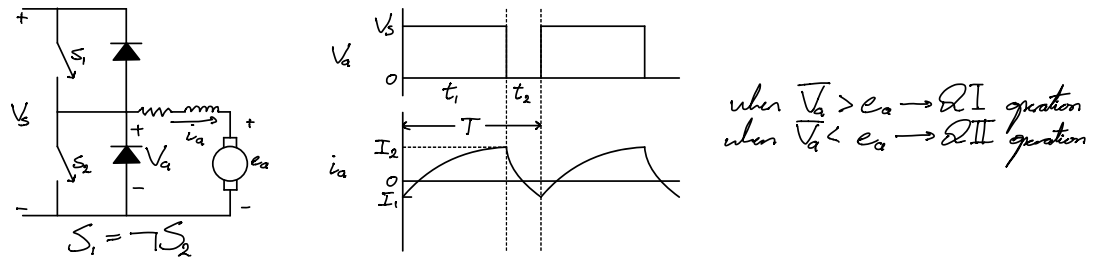
- DC-DC Converter Quadrants
 - each quadrant represents a different mode of operation



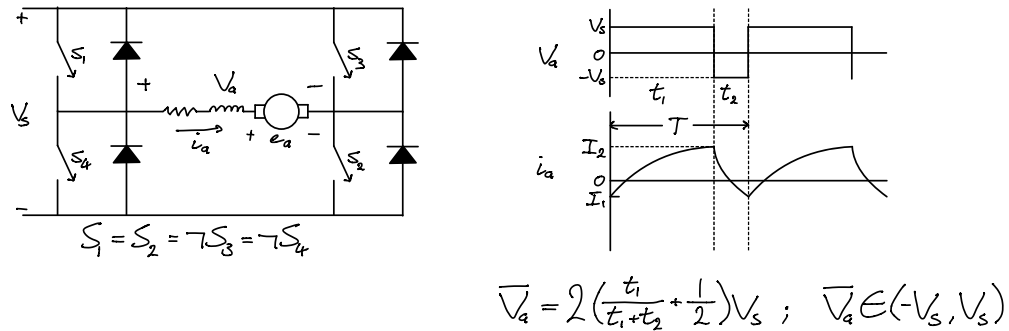
• PWM — Quadrant I operation



• PWM — Quadrant I/II operation

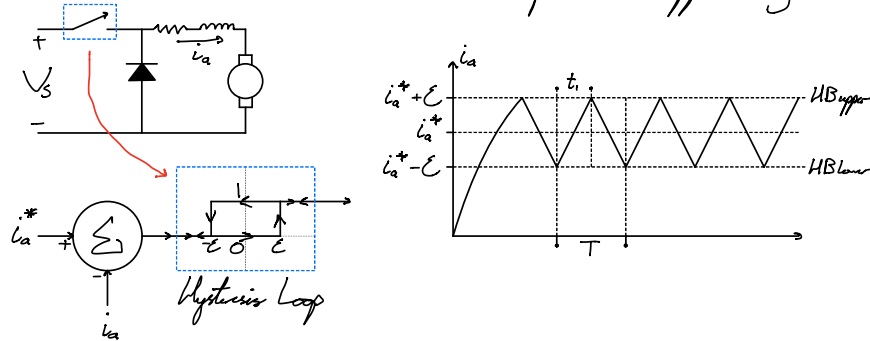


• PWM — Quadrant I-IV operation



• Current Source DC-DC Converter

- Hysteresis Current Control: real time current (i_a) switches between upper/lower hysteresis bands (HB) to provide an approximately stable current (i_a^*)

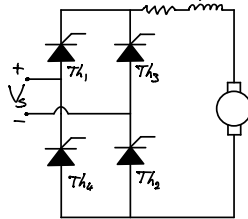


Thyristor Motor Drive

- a thyristor operates as a diode / open circuit depending on gate current



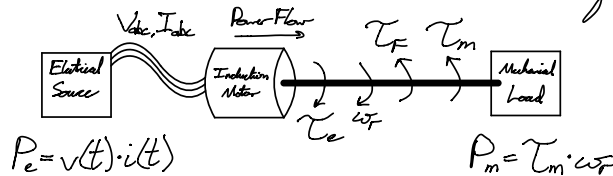
Fully Controlled Rectifier



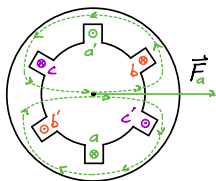
- current passes through TH_1 & TH_2 during *positive* voltage wave, and through TH_3 & TH_4 during *negative* voltage
- Thyristors allow current flow when *forward biased*

Induction Machines

- machine with AC current in rotor and stator windings



- Wound Rotor Winding: three phase connection through rotor allowing double feed as well as both generator / motor capability
- Rotating Magnetic Field from Stator

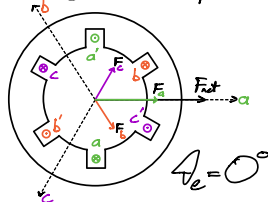


- each winding (i.e.: a & a') generates magnetic field

$$F_{rt} = N_{im} \cos(\omega_e t) + N_{im} \cos(\omega_e t - 120^\circ) + N_{im} \cos(\omega_e t + 120^\circ)$$

F_a F_b F_c

- at any time t , phasor sum of F_a , F_b , F_c equals F_{rt} :



$$\vec{F}_{rt} = \vec{F}_a + \vec{F}_b + \vec{F}_c = \frac{3}{2} N_{im} \angle \theta_e$$

- reordering phases (phase angle order) can reverse rotation direction
i.e.: $a, b, c \rightarrow b, a, c$

- Slip — Asynchronous Speed
 - difference between speed of rotor and stator's magnetic field

$$s = \frac{\omega_e - \omega_r}{\omega_e}$$

- Slip Frequency: speed of rotor relative to stator's magnetic field

$$\omega_s = s \cdot \omega_e$$

- Synchronous Speed: speed at which magnetic field rotates

$$\omega_{syn} = \frac{2}{p} \omega_e$$

of poles

- convert a P-pole motor to an equivalent 2-pole motor:

$$\omega_{2p} = \frac{p}{2} \omega_{rm}$$

2-pole equivalent rotor speed
actual rotor speed

- Rotor Speed: $\omega_{rm} = (1-s)\omega_{syn}$; $s=1$ on startup $\rightarrow \omega_r = 0 \text{ rad/s}$

- Rotor Winding Current Frequency: $f_r = s \cdot f_e$

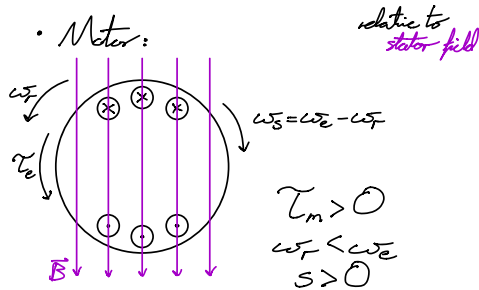
- Principle of Operation

- Idle: $s=0$

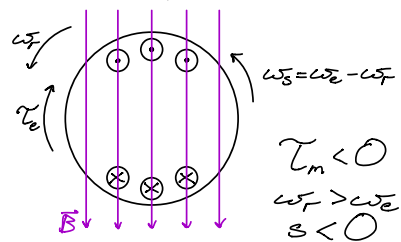
- zero magnetic flux crosses rotor conduction bars $\rightarrow 0$ torque produced

$$\omega_r = \omega_e \quad \tau_e = 0$$

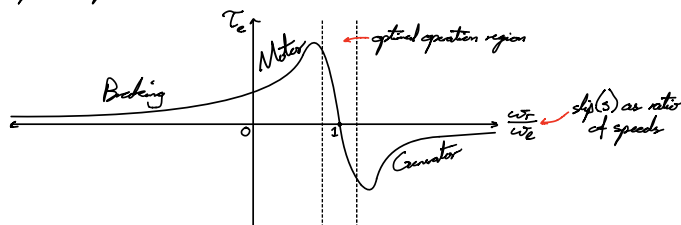
- Motor:



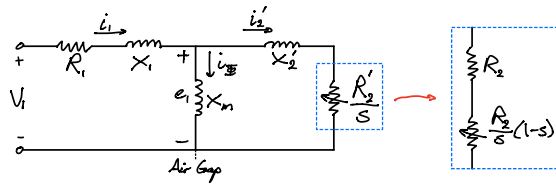
- Generator:



- Torque-Speed Characteristic



• Equivalent Circuit



$$e_1 = e_2 a$$

$$i_2' = \frac{i_2}{a}$$

$$X_2' = X_2 \cdot a^2$$

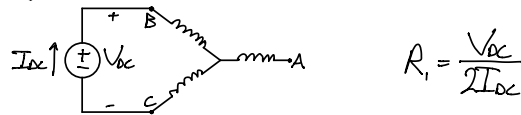
$$\frac{R_2'}{s} = \frac{R_2 a^2}{s}$$

• Machine Parameters

- determine induction motor equivalent parameters through tests

1. DC Measurement of Winding Resistance (R_1)

- apply DC voltage across two stator connections



$$R_1 = \frac{V_{dc}}{I_{dc}}$$

2. No Load Test

- Rotational Losses: $P_{rot} = P_{NL} - 3R_1 I_{NL}^2$
- Reactance: $X_1 + X_m = X_{NL} = \sqrt{Z_{NL}^2 - R_{NL}^2}$

3. Blocked Rotor Test

- Resistance: $R_1 + R_2' = \frac{P_{BR}}{3I_{BR}^2} = R_{BR}$
- Reactance: $X_1 + X_2' = \sqrt{Z_{BR}^2 - R_{BR}^2}$

• Power Conversion

$$P_c = 3I_2'^2 \frac{R_2'}{s} (1-s) = \tau_e \cdot \omega_{rm}$$

• Maximum Torque

- occurs at certain values of slip: $s_{c_{max}} = \pm \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}}$

Thermin Equivalent
Parameters

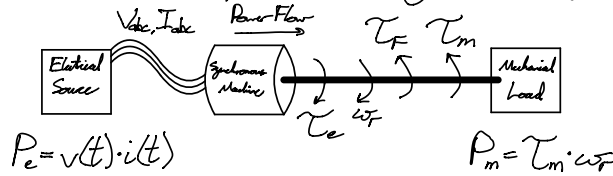
- upon motor startup, the value of slip is 1

• Speed and Torque Control

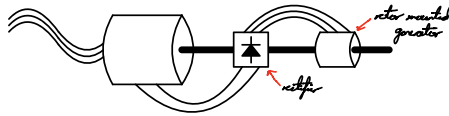
- control by changing circuit values:
 1. Line Voltage
 2. Rotor Resistance
 3. Frequency $\rightarrow$ Variable Frequency Drive

Synchronous Machines

- speed of rotor matches speed of rotating stator magnetic field: $\omega_r = \omega_e$

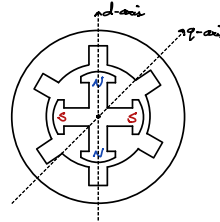


- Brushless Excitation: using a rotor mounted generator and rectifier to produce DC current for field winding rotor field



- Permanent Magnet Excitation: use magnets to establish rotor field
- Direct (d) and Quadrature (q) axes

- d-axis represents the direction of field magnetic flux (air gap flux)
- q-axis represents the direction of mmf due to armature current

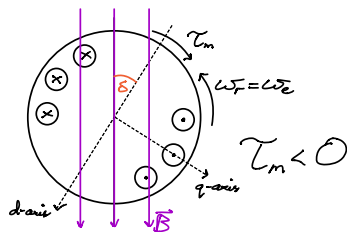


- Principle of Operation

- Idle

- rotor poles aligned with stator poles $\omega_r = \omega_e$ $T_e = 0$
- $\omega_r = \omega_e$ $T_e = 0$

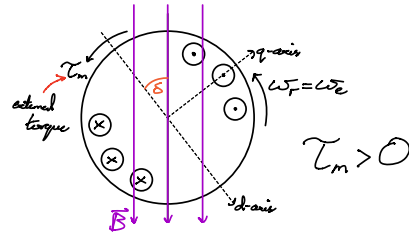
- Motor:



- rotor lags stator field
- external torque is direction of rotation: $\hat{T}_m = -\hat{\omega}_r$

- Rotor Angle: $\delta = \theta_r - \theta_e = \int \omega_r - \omega_e dt$

- Generator

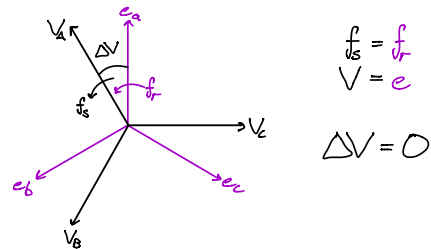


- rotor leads stator field
- external torque opposes rotation direction: $\hat{T}_m = \hat{\omega}_r$

- Startup
 - Variable Frequency Drive motor:
 1. generate rotor field by applying voltage to field winding
 2. slowly apply stator current to increase ω_e as well as maintain synchronization
 - start as Induction Motor
 - bring rotor speed up to ω_e while unloaded, then apply voltage to field winding

- Connecting Synchronous Machine

- sequence of phases, frequency, phase difference, and voltage magnitudes between machine and power grid must match to safely connect

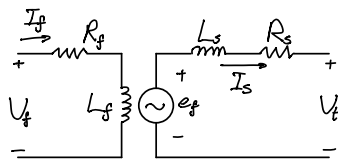


- Infinite Bus

- power supply grid large enough to be unaffected by change in load
- while synchronous machine is connected to infinite bus, excitation voltage (e_f) changes linearly along the modified air gap line:

$$e_f = \frac{\Delta V}{\Delta I} \cdot I_f$$

- Equivalent Circuit

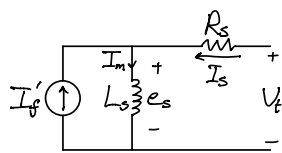


$$V_t = \pm (R_s + jX_s)I_s + e_f$$

$$X_s = j\omega_e L_s = X_{ms} + X_{ls}$$

stator leakage reactance
 stator magnetizing reactance

- represent back emf and stator winding with Norton Equivalent:
 - useful for constant field current conditions



$$I_f' = \frac{e_f}{X_s}$$

$$|I_f'| = \frac{X_{ar}}{X_s} n I_f$$

$$n = \frac{\sqrt{2}}{3} \frac{N_{re}}{N_{se}}$$

- Power Angle (δ): angle between e_f and V_t

- positive in generators
- negative in motors

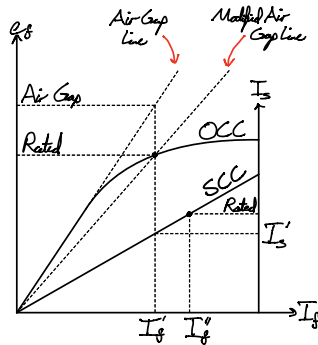
Parameter Determination

- use DC source to measure winding **resistances**:

$$R_s = \frac{V_{dc}}{2I_{DC}}$$

$$R_f = \frac{V_{dc}}{I_{DC}}$$

- Open Circuit and Short Circuit tests to determine **reactances**:



$$Z_{s \text{ unsaturated}} = \frac{E_g A_g}{I'_s}$$

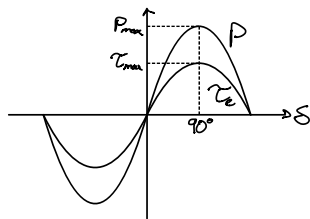
$$X_s = \sqrt{Z_s^2 - R_s^2}$$

$$Z_{s \text{ saturated}} = \frac{E_{g \text{ rated}}}{I'_s}$$

- Short-Circuit Ratio: $SCR = \frac{I'_s}{I_{s \text{ rated}}}$

Power and Torque Characteristics

- effects of δ in constant voltage and speed conditions:



$$P = 3V_t I_s \cos \phi = 3 \frac{V_t E_g}{X_s} \sin \delta = \omega_{syn} T_e$$

$$P_{max} = 3 \frac{V_t E_g}{X_s} = \omega_{syn} T_{max}$$

- Static Stability Limit ($\delta = 90^\circ$)
 - machine can be loaded **gradually** up to the torque and power limit before losing synchronism

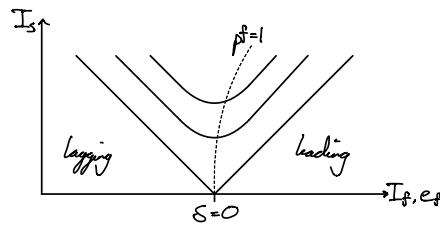
Reactive Power

$$Q = 3V_t I_s \sin \phi = 3 \frac{V_t E_g \cos \delta - V_t^2}{X_s}$$

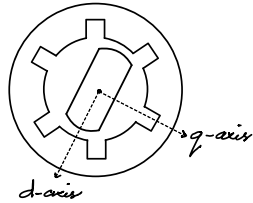
- by changing I_f , one can achieve:

- Capacitive Load (leading pf): $Q > 0$
- Inductive Load (lagging pf): $Q < 0$

- Synchronous Condenser
 - *unloaded* synchronous machine used to add reactive power to grid



- Salient Pole Machines
 - allows for easier placement of rotor windings, when using many poles
 - reactance of machine depends on angle between rotor and *quadrature* / *direct* axes

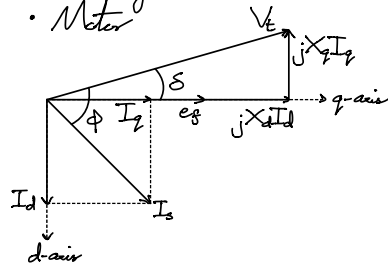


$$I_s^2 = I_q^2 + I_d^2 = [I_s \cos(\phi \pm \delta)]^2 + [I_s \sin(\phi \pm \delta)]^2$$

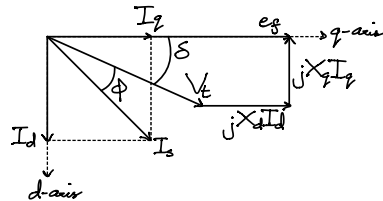
$$e_f = V_t + R_s I_s + jX_q I_q + jX_d I_d$$

$$\tan \delta = \frac{I_s X_q \cos \phi}{V_t \pm I_s X_q \sin \phi}$$

- Phasor Diagrams
 - Motor



- Generator



- Power Transfer

- power due to excitation voltage (e_f): $P_f = \frac{|V_t| |e_f|}{X_d} \sin \delta$

- power due to reluctance torque of saliency: $P_r = \frac{|V_t|^2 (X_d - X_q)}{2X_d X_q} \sin 2\delta$