Interest & Equivalence

- the same sum of money is valued differently depending on when it is available
- Time Value of Money
  - money can be rented (interest) similar to a commodity
  - Present Value of Money: measures current worth of future benefits/costs

$$PV_n = \frac{\$_n}{(1+r)^n}$$

  - Simple Interest: interest that is only calculated on the original sum

$$F = P(1 + i \cdot n)$$

F: future sum; P: present sum; n: # of time periods; i: interest per time period

  - Compound Interest: interest calculated on accumulated sum

$$F = P(1+i)^n$$

  - Repaying a Debt: multiple methods of repaying a loan in installments
    1. Constant Principle: pay constant portion of loan plus total interest
    2. Interest Only: pay only total interest
    3. Constant Payments: pay exact same amount for each installment
    4. Lump Sum: pay all money back at end of term

- Equivalence: with an appropriate interest rate, all 4 methods of loan repayment are equivalent
- Interest Rates
  - the interest rate usually differs from the frequency of compounding
  - Nominal: only considering the interest rate
  - Effective: the interest rate converted to the period of the frequency of compounding

$$i_e = \frac{r}{m}$$

r ← declared rate; m ← # compounding periods within "r" timeframe

  - Effective Annual: effective rate at annual period

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

Equivalence for Repeated Cash Flows

- costs, savings, revenues now and in the future
- Opportunity Cost: value of the next best foregone alternative
  - a cost measured in terms of alternative goods/services
- Uniform Series Compound Interest Formulas

$$F = A\left[F/A, i\%, n\right] = A\cdot\left[\frac{(1+i)^n-1}{i}\right] = P\left[F/P, i\%, n\right] = P\cdot\left[1+i\right]^n$$

(F: future sum; A: repeating amount)

$$P = A\left[P/A, i\%, n\right] = A\cdot\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] = F\left[P/F, i\%, n\right] = F/\left[1+i\right]^n$$

(P: present value)

$$A = F\left[A/F, i\%, n\right] = F\cdot\left[\frac{i}{(1+i)^n-1}\right] = P\left[A/P, i\%, n\right] = P\cdot\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

- Arithmetic Gradient
  - when repeating amount (A) increases by a uniform repeating amount (G):
    - separate present value (P) into uniform (P') and increasing (P'') components:

$$P = P' + P'' = A\cdot\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] + G\cdot\left[\frac{(1+i)^n-i\cdot n-1}{i^2(1+i)^n}\right]$$

$$A = G\left[A/G, i\%, n\right] = G\cdot\left[\frac{1}{i} - \frac{n}{(1+i)^n-1}\right]$$

  - when repeating amount (A) increases by a uniform repeating rate (g):

(A_1: first payment)

$$P = A_1(P/A, g\%, i\%, n) = A_1\cdot\left[\frac{1-(1+g)^n(1+i)^{-n}}{i-g}\right];\ i\neq g$$

$$= A_1\cdot\left[n(1+i)^{-1}\right];\ i=g$$

- Continuous Compounding at rate r:

$$\left[A/F, r\%, n\right] = \frac{e^r-1}{e^{rn}-1} \qquad \left[A/P, r\%, n\right] = \frac{e^{rn}(e^r-1)}{e^{rn}-1}$$

$$\left[F/A, r\%, n\right] = \frac{e^{rn}-1}{e^r-1} \qquad \left[P/A, r\%, n\right] = \frac{e^{rn}-1}{e^{rn}(e^r-1)}$$

Present Worth Analysis

- to compare cash flows of possible solutions, their economic impact must be analysed at the same point in time
- depending on lifespan of solutions, different time periods of analysis can be chosen
  - if lifespan of alternatives differ, use LCM of lifespans for comparison
- Capitalized Costs: present sum required to operate solution indefinitely
  - requires that the pirncipal not be spent, but the interest earned can be
- Future Worth Analysis: determining equivalent value of solutions at a future date

Annual Cash Flow Analysis

- Equivalent Uniform Annual Cost
  - resolve any series of cash flows into an annual equivalent
  - Salvage Value: one time cash flow benefit at end of asset's life
    - net benefit of such an asset is: $EUAC = P(A/P, i, n) - S(A/F, i, n)$ ← salvage value
  - when comparing alternatives, unequal lives has no effect
- Mortgage: loan for purchase of real property with a lien on property serving as security
  - Amortization: paying off debt over time
  - interest rates can be fixed term or variable and usually depend on the bond market
  - Equity: value of property minus all loans/mortgages

Rate of Return Analysis

- determine interest rate that allows for certain cashflows/assets
- Internal Rate of Return: interest rate so that net equivalent value equals zero
  - Incremental IRR: the difference in IRR between current choice and alternative
- Return On Investment: percent change of investment over a set period

$$ROI = \frac{F-P}{P}$$

- Minimum Attractive Rate of Return: lower bound of IRRs for project to be attractive

- Sensitivity Analysis: compare alternatives at various interest rates to choose best one
  - variable interest rates is typically a continuous range

Benefit-Cost Ratio Analysis

- quantify a project to show whether it causes a net increase in economic benefit

$$\text{conventional B/C ratio} = \frac{\text{Equivalent Worth of Benefits}}{\text{Equivalent Worth of Costs}} = \sum \frac{TB_n}{(1+r)^n} \Big/ \sum \frac{TC_n}{(1+r)^n}$$

$$\text{modified B/C ratio} = \frac{\text{Equivalent Worth of Benefits} - \text{Ongoing Costs}}{\text{Initial Costs}}$$

- Payback Period: time for profit/benefits to equal cost
- Cost Effective Analysis: how to meet predetermined goal most cost-effectively

MARR Selection

- lower bound of MARR is cost of money borrowed to invest in project
- Risk: if probabilities of future outcomes are known
- Uncertainty: if probabilities themselves are unknown
- projects with normal risk & uncertainty use the MARR, whereas projects with greater risk & uncertainty use increased MARR

Public Sector Analysis

- investment decisions consider different factors in public sector than in private
- Scope of Analysis
  - project stakeholders usually involve certain levels of government
  - projects often have longer lifespans (20-50 years)
- Financing
  - governments often use tax and bonds as source of capital
- Opportunity Cost
  - Government: interest rate set at that of best prospective project
  - Taxpayer: interest rate set at what taxpayer could have received through other means

Measuring Benefits

- Market Values: measure market benefits
- Non-Market Values
  - Indirect Use: measure what people do in some related way
  - Non-Use Values (passive use)
    - Option Value
    - Bequest Value
    - Existence Value

- Complementary Method: measure people's expenditure on goods associated with environmental change
- Travel Cost Method: measure travel costs people endure to infer value of project
- Hedonic Price Method: measure benefits that people get from environmental goods
- Foregone Earnings Method: measure difference in wages in different locations of similar job
- Averting Behaviour Method: measure people's expenditure on goods associated with environmental improvement
- Value of Statistical Life: cost everyone is willing to pay to save 1 human life
- Contingent Valuation Method: measure what people are willing to pay to keep a good

Inflation

- difference between change in dollar amount income, and purchasing power

- Inflation Rate: f
- Real Interest Rate: i' $\longrightarrow$ $i' = \frac{i-f}{1+f}$
- Market Interest Rate: i

- Actual Dollars: value that carries the effect of inflation
- Real Dollars: constant dollars representing purchasing power of a base year

- Price Index: describe the relative price fluctuation
  - % increase(n) $= \frac{\text{Index}(n) - \text{Index}(n-1)}{\text{Index}(n-1)}$

Uncertainty

- Probabilities
  - probabilities remain between 0 & 1 and total to 1
  - Joint Probability: $P(A \text{ and } B) = P(A) \cdot P(B)$
  - Expected Value: $\sum_{i=A} \text{Outcome}_i \cdot P(i)$

- Risk: chance of getting outcome other than EV
  - Standard Deviation: $\sigma = \sqrt{EV(X-\mu)^2}$
  - Rule of Thumb: project is safe if expected present worth is at least double standard deviation of present worth

Accounting

- 3 Functions within Business
  1. Engineering Economics: analyze economic impact of projects over their life cycles
  2. Management: finds and allocates resources in firm
  3. Accounting: reports on transactions, financial state, and impact of past decisions

- Balance Sheet
  - Assets = Liabilities + Equity
    - Assets: all things owned by firm that have value
    - Liabilities: all money firm owes to others
    - Equity: value of firm
  - Current Assets: less than 12 months to liquidate
  - Long Term Assets: difficult to liquidate in short term
- Net Worth Statement: reports firm's assets at one point in time
- Income Statement
  - summary of firm's revenues & expenses over period of time
  - Operating Expense: cost that can be "turned off"
- Cash Flow Statement: portrays liquidity of firm
  - Working Capital = Current Equity
- Cost Accounting: method of assigning costs to products
  - Traditional: assign direct & indirect costs to activities for product
  - Activity-Based Costing: activities are linked to products; more direct costs

- Ratio Analysis
  1. Liquidity: measure of company's ability to pay off debt
     - quick ratio = $\frac{\text{current assets} - \text{inventory}}{\text{current liabilities}}$, interest coverage ratio = $\frac{\text{total income}}{\text{interest payments}}$
  2. Solvency: relationship between debt and equity financing
     - debt to equity ratio = $\frac{\text{total liabilities}}{\text{owner equity}}$, equity to asset = $\frac{\text{owner equity}}{\text{total asset}}$
  3. Profitability: compare business earnings against all economic resources
     - return on equity = $\frac{\text{net income}}{\text{average owner equity}}$

Income, Depreciation, Cash Flow

- Depreciation: non-cash cost that causes real cash impacts
  - asset must:
    1. be used for business purposes
    2. have a deterministically useful life that is longer than one year
    3. decay/lose value to
  - can be used to claim business expenses
  - Book Value = Cost Basis - Depreciation Charges
  - Straight-Line Depreciation:

$$d_t = \frac{B - S}{N}$$

(B: Cost Basis, S: Salvage Value, N: Depreciable Life)

  - Sum-of-Year's-Digits Depreciation:

$$d_t = \frac{N - t + 1}{N(N+1)} \cdot 2(B - S)$$

(t: Year)

  - Declining-Balance Depreciation

$$d_n = D \cdot B(1 - D)^{n-1}$$

(D: constant Depreciation Rate)

  - Unit-of-Production Depreciation

$$d = \frac{\text{Production of Year}}{\text{Total Lifetime Production}} (B - S)$$

- Capital Cost Allowance — Depreciation
  - First Year Depreciation: $CCA_1 = P \cdot \frac{d}{2}$ (P: Initial Value)
  - Subsequent Depreciation: $CCA_n = P \cdot \frac{1-d}{2}(1-d)^{n-2}$

- Asset Disposal — Depreciation
  - Recaptured CCA: asset sold for more than book value; results in extra taxable income
  - Loss on Disposal: asset sold for less than book value; results in a loss
  - Capital Gains: asset sold for more than original cost; results in excess being taxed at lower rate

After-Tax Cash Flows

- Taxable Income for Individuals
  - Average Tax Rate: total taxes payable / taxable income
  - Marginal Tax Rate: tax rate that applies to next taxable dollar
  - Credit: money returned from tax agency
  - Exemption: money not taxed
- Accounting & Engineering Economy
  - Before-Tax Cash Flow = Operating Revenue - Operating Costs
  - Taxable Income = BTCF − CCA − debt Interest
  - Net Profit = Taxable Income − Income Tax
- Handling Asset Purchases
  - present worth of after-tax cost of depreciable capital asset is:

$$PW = P\left(1 - \frac{td}{i+d} \cdot \frac{1+i/2}{1+i}\right)$$

(td: CCA rate; i: discount rate)

  - present worth of after-tax cost of salvage of disposable asset at end of n terms:

$$PW = S\left(1 - \frac{td}{i+d}\right)$$

(td: tax rate)

- Working Capital: money to cover time between when costs are incurred, and when revenues are received
- Estimating After-Tax Rate of Return
  - when non-depreciable assets are lump-sum paid at end:

$$\text{After Tax RoR} = (1 - \text{Marginal tax rate}) \cdot (\text{Before Tax RoR})$$

Sources & Costs of Borrowed Funds

- Sources of Capital
  1. Money Generated from Firm's Operation
  2. Money Borrowed
  3. Money from Selling Stock
- Cost of Capital: Weighted Average Cost of Capital

$$\text{WACC} = [(\text{1st fraction of total capital}) + (\text{2nd fraction of total capital}) + \ldots] \cdot \text{RoR}$$

Replacement Analysis

- reasons for asset replacement
  1. Obsolescence
  2. Aging Deterioration
  3. Depletion
- Minimum-Cost Life of New Asset
  - # years that minimizes EUAC

1. defender's marginal costs can be computed and are increasing
   - keep existing asset as long as MC of ownership is less than minimum EUAC of challenger
2. defender's marginal costs can be computed and are NOT increasing
   - replace when defender's increasing MC exceeds challenger's minimum EUAC
3. defender's marginal cost unavailable
   - compare defender's EUAC to challenger's minimum EUAC